Testing the Gaussian Legendre Quadrature Method for Surface Flux Accuracy

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In order to confidently predict climate change, a deep understanding and accurate numerical representation of atmospheric solar radiative transfer is essential. As solar radiation travels from the top of the atmosphere (TOA) to the surface, atmospheric constituents such as clouds, aerosols, and greenhouse gases can scatter or absorb solar radiation, thereby modulating surface solar irradiance. The energy reaching the surface is partially absorbed, leading to warming, and is later dissipated through longwave emission, general circulation of the atmosphere and oceans, and chemical and biological processes. While TOA fluxes can be inferred directly from satellite observations, surface shortwave (SW) fluxes exhibit larger uncertainties, reducing confidence in their applications.

These uncertainties arise, in part, because radiative transfer models (RTMs) are required to compute surface SW fluxes in conjunction with satellite observations. RTMs employ multiple approaches for calculating SW fluxes, including line-by-line (LBL), analytic and Monte Carlo solvers, and band and correlated k-distribution (CKD) models. The choice of model involves trade-offs that affect computation time depending on spectral resolution and the number of spectral bins. Faster computations allow for more extensive analyses but at the expense of spectral detail.

For global calculations, computational power is further constrained by spatial resolution, as RTM calculations are required for each grid cell. The NASA Cloud and Earth Radiant Energy System (CERES) uses a 1° latitude × 1° longitude spatial resolution for its global gridded products. While this limits excessive computations, representing the full atmospheric and surface variability within a one-dimensional RTM remains challenging. Clouds, for example, can vary substantially in cloud optical depth (COD) and liquid water path (LWP) within a single grid cell. Using the mean or median of these cloud properties does not yield accurate results because radiative interactions with clouds are nonlinear. Therefore, alternative approaches are needed.

This proposal seeks to apply a Gaussian–Legendre Quadrature (GLQ) approach to address this issue. The GLQ method was developed by Li and Barker (2018) to compute domain-averaged radiative fluxes at the surface and TOA to test within climate models. In this method, LWPs are sorted from smallest to largest for each sub-column within the domain. The number of Gaussian quadrature nodal positions (n_G = 1, 2, or 3) is selected, with each position corresponding to a different sorted LWP. The RTM is run for each nodal position, and the resulting fluxes are averaged. As the number of nodal positions increases, RTM flux errors decrease because more cloud variability is represented. However, to limit computational burden, the smallest number of nodal positions should be selected without introducing substantial error. Figure 1 demonstrates the GLQ method from Li and Barker (2018). In each of the three panels, sorted LWPs and increasing nodal points are shown. The domain-averaged flux is plotted above. Using n_G equal to the total number of sub-columns provides a benchmark value of 341.6 W m^{-2}. As shown in Figure 1, increasing the number of nodal points yields a mean flux closer to this benchmark. Although Li and Barker (2018) evaluated the GLQ method using climate model output, it has not yet been applied to real-world observations.

This research will incorporate the Canadian Centre for Climate Modelling and Analysis (CCCma) CKD RTM. Previous work by Brendecke et al. (2024, 2025, 2026) demonstrated that this RTM performs exceptionally well despite its low spectral resolution. Comparisons against surface and TOA measurements showed errors comparable to those from the NASA Fu–Liou RTM used in the SYN1deg product. Inputs to the CCCma model will be derived from MODIS and VIIRS satellite cloud retrievals, including COD, particle effective radius (Re), and cloud base and top heights. The pixel resolution of these retrievals is approximately 1 km; within a 1° × 1° grid cell, this corresponds to roughly 11,000 sub-columns. Additional inputs will include atmospheric profiles of water vapor, temperature, and ozone from the MERRA-2 reanalysis product. Aerosol optical depth and aerosol type will also be retrieved from MODIS and VIIRS. Finally, surface spectral albedos will be obtained from data provided by the International Geosphere–Biosphere Programme (IGBP) based on identified surface type.

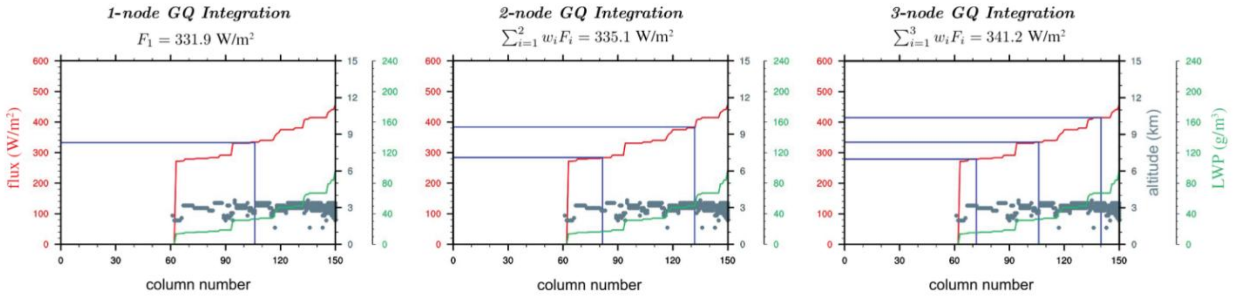

Figure 1. 150 stochastically generated sub-columns for a domain. Sub-columns were sorted based on their cloud liquid water paths, LWP (green lines). Dots represent cloud-bearing sub-columns. Reflected SW fluxes at TOA for each sub-column are shown by red lines. GQ integration columns, and their respective fluxes, for n_G =1, 2 and 3 are shown by blue lines. Estimated domain-average reflected SW fluxes are listed above each plot. A benchmark of 341.6 W/m^2 is calculated using all sub-columns (Li and Barker, 2018).

Analysis of the GQ-ICA method will incorporate two approaches to quantify changes in error. The first approach involves running the RTM for every sub-column within the domain and computing the average flux. Calculating fluxes at each column accounts for all cloud property variability and quantifies spatial uncertainty. This method will also help determine the minimum number of nodal points required to approximate the full sub-column benchmark. Although this approach provides the most accurate calculations, it is computationally expensive for operational applications and is therefore limited to research testing.

To further evaluate the method, flux observations will be used. Observations provide the most accurate estimates of radiative fluxes; however, due to their limited spatial coverage, they cannot be used to derive global averages. By aligning model domains with observation sites, comprehensive model error quantification can be achieved. The Southern Great Plains (SGP) site provides an ideal test location. Within its 1° spatial domain, surface properties are relatively homogeneous, and seven observation sites measure fluxes to capture spatial variability. Performing both analyses will allow for quantification of spatial and model uncertainties.

The project timeline is approximately one year. Because previous work has already evaluated the CCCma RTM under different sky conditions, model setup will be straightforward. Most of the data processing time will be devoted to testing the GLQ method. Although the conceptual framework is straightforward, implementation will be more complex for real-world cases; specific methodologies for applying the GLQ method to multiple cloud layers and mixed-phase clouds will need to be developed. Once established, ten years (2015–2025) of cases will be analyzed at the SGP site, with additional sites included if necessary. Model results will be compared with observations, and statistical error analyses will be performed. These steps are expected to take approximately six months. Following completion of the analysis, a manuscript will be prepared and submitted for publication, requiring an additional six months.

References:

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